

## Normative Decision Making with Multiattribute Performance Targets

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### ABSTRACT

Many companies set multiple performance targets for their managers and reward them on meeting a threshold value for each target or goal. Examples of such incentive structures abide in the managerial literature and in organizational settings. We show that this incentive structure, while popular, has two main problems: (i) it can induce managers who try to maximize the probability of meeting their performance targets to make decisions that are not compatible with expected utility maximizing decisions, and (ii) it may lead to trade-offs among the performance objectives that are inconsistent with the corporate value function. In this paper, we propose a method to remedy these two problems, while retaining a target-based incentive scheme. We define a multiattribute target as a deterministic region in the space of multiattribute outcomes that has two properties: (1) the probability that the outcome of a multiattribute lottery lies within the target region is equal to the expected utility of the lottery, and (2) all outcomes within the target region are preferred to all outcomes outside it. These two properties lead to a new quantity; which we call the ‘value aspiration equivalent’ that leads managers who maximize the probability of meeting their targets to simultaneously maximize the expected utility, and it also induces trade-offs that are consistent with the decision maker’s value function. Copyright © 2009 John Wiley & Sons, Ltd.

KEY WORDS: aspiration equivalent; performance targets; multiattribute utility

### 1. INTRODUCTION

Expected utility maximization is a rigorous approach to ranking and selecting the best decision alternative. From a descriptive view, however, individuals often prefer to simplify their daily decision-making activities by setting goals for themselves to provide focus and motivation. In a deterministic setting, individuals then choose the alternatives that meet their specified goals, and in a probabilistic setting, they choose the alternatives that have the highest probability of meeting (or exceeding) these goals. This behavioural simplification was discussed in the classic work of Simon (1955) in his introduction of aspiration levels and satisficing. Simon illustrated how an aspiration

level divides the domain of any attribute into two regions: the region to the right of the aspiration level, which is considered satisfactory (or the target region) and the region to the left, which is considered unsatisfactory (Figure 1). We can think of a person operating with the notion of satisficing as one who has a step utility function that jumps at the aspiration level.

The simplicity of conceptualizing only two possible outcomes (satisfactory and unsatisfactory), has extended the notion of satisficing to organizational settings and has been popularized under what is commonly known as target-based incentives using tools such as Management by Objectives (MBO) and Balanced Scorecards (Drucker, 1993; Olson, 1968; Humble, 1971; Carrol and Tosi, 1973; Mossholder, 1980; Kaplan and Norton, 1996). Under these incentive structures, a manager is given a set of performance objectives and is rewarded on meeting specified target levels for these objectives or goals. This multi-objective incentive structure defines a

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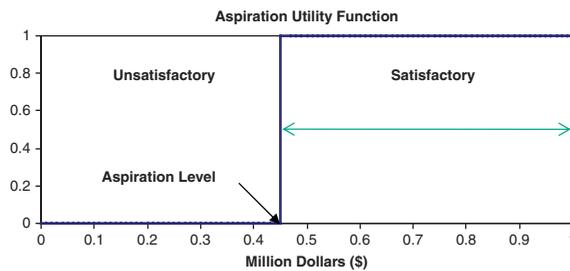


Figure 1. Aspiration (step) utility function showing satisfactory and unsatisfactory regions.

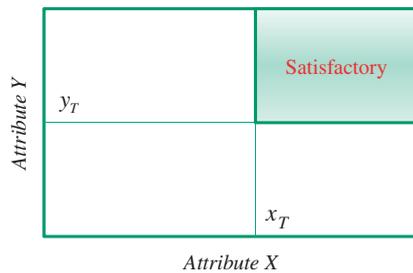


Figure 2. Rectangular target region with two attributes.

rectangular region as the target region (for two attributes) and a multidimensional hyper-rectangular region when more objectives are present. Figure 2 shows an example of a target region defined by threshold values for two objectives  $X$  and  $Y$ , with target values  $x_T$  and  $y_T$ , respectively.

As we shall see, however, the extension of Simon's notion of satisficing to decisions with multiple attributes is not to set multiple independent aspirations levels for each attribute (or goal). Similarly, target-based formulations that consider multiple attributes should not focus on setting independent target levels for each objective. When uncertainty is present, two fundamental problems arise with setting a fixed threshold value for each objective and then choosing the alternative that has the highest probability of meeting (or exceeding) these threshold values. First there is no guarantee that this choice is consistent with the expected utility maximizing alternative (von Neumann and Morgenstern 1947). Second, as we shall see, setting specified threshold levels for each objective may cause trade-offs that are inconsistent with the decision maker's (or the corporate) value function. These two problems may explain a recent article in the Wall Street Journal [Hymowitz (2005)] that states '*targets set by upper management, as well*

*as higher-level targets related to stockholders' expectations about various measures, provide incentive for good performance but also can create stress and perhaps lead to the types of ethical problems that have come to light in recent years*'. A fundamental requirement for a given target-based incentive should be that a manager who tries to achieve his given target is induced to make decisions that maximize the organization's expected utility. Targets must be chosen carefully, and in a specific manner, to achieve the normative utility-maximizing behaviour and must also induce trade-offs that are consistent with the value function.

In this paper, we present a method to set multiattribute targets that remedies these two problems. We define a *target* as a deterministic region in a multiattribute outcome space. When an outcome falls within it, it is considered satisfactory, or a 'success', and when it falls out of it, it is considered unsatisfactory, or a 'failure'. We then define *normative targets* as ones that induce decision makers, who choose their alternatives in a way that maximizes their chance of achieving their target, to select the alternatives that have the highest expected utility. While we motivate this target-based formulation in a managerial setting, where the corporation and the manager have the same information about the different projects, we observe that this formulation works just as well for personal decision making where an individual sets multiattribute threshold targets for himself. Setting individual multiattribute targets may lead to decisions inconsistent with value trade-offs unless they are set in accordance with the individual's value function (see Dyer and Sarin, 1982 and Matheson and Abbas, 2005 for more information on value functions).

To assure that the target region is consistent with normative decision making and with the trade-offs provided by the decision maker's value function, we require that each outcome within the target region be preferred to all outcomes outside it. We call a target region meeting this condition a value-based target. This is our first criterion, which is a meaningful criterion for any target, otherwise an outcome that is labelled 'satisfactory' and falls within the target region would actually be less preferred to one that is labelled 'unsatisfactory' and falls outside the target region. Assuming that the multiattribute value function is sufficiently smooth to define contours of constant value, we will see that this condition leads to a characterization of a multiattribute target region whose boundaries lie on an isopreference

contour of the decision maker’s value function (or possibly several disconnected target regions if the utility function has multiple peaks). Our second criterion is that the probability that the outcome lie within this target region be equal to the expected utility of the lottery (we require that the utility function be normalize to have a range in the interval [0,1] and so the expected utility also lies within this interval). We call this criterion the normative utility criterion. This criterion makes the alternative with the highest probability of achieving its target region be (simultaneously) the alternative with the highest expected utility. We then show how to determine the contours of the value function that meet both the normative and the value-based target setting criteria.

## 2. REVIEW OF PREVIOUS WORK

In our search of the literature, we have found a line of work relating expected utility decision making to target-based decision making when only single attributes are present. Some of this literature focuses on setting uncertain targets for the decision maker. For example, (Borch, 1968) uses a normalized utility function,  $U(x)$ , from which he defines a cumulative distribution function over loss,

$$F_1(-x) \triangleq 1 - U(x) \quad \text{or}$$

$$F_1(y) \triangleq 1 - U(-y)$$

Borch then describes his model as follows; ‘Let us assume that our decision maker, independently of the decision to be taken, already is exposed to a risk which can lead to a loss  $y$ ’. He shows that choosing the lottery with the highest expected utility is equivalent to choosing the lottery that minimizes the probability of ruin (the outcome of the chosen lottery plus the negative utility outcome,  $y$ , falls below zero—an ‘unsatisfactory’ region). Borch clearly undertook this interpretation in order to create an ‘aspiration of survival’, and later develops methodology for viewing sequences of gambles.

Castagnoli and LiCalzi (1996) provide a different interpretation for the normalized utility function,  $U(x)$ , and think of it as a sampling distribution from which an independent uncertain target,  $T$ , is generated. A decision maker who makes choices by maximizing the probability of meeting this uncertain target is also consistent with expected utility maximization. To relate this interpretation to Borch’s interpretation, observe

that beating the generated target  $T$  is equivalent to subtracting the generated target from the obtained lottery outcome and beating zero. The probability that a performance,  $X$ , with cumulative distribution function,  $F(x)$ , exceeds an uncertain target,  $T$ , generated by a distribution,  $U(x)$  is

$$P(X > T) = 1 - \int_{t=-\infty}^{\infty} F(t) dU(t) \tag{1}$$

where  $t$  is a dummy variable. Using the rule of integration by parts for a normalized utility function gives

$$P(X > T) = \int_{t=-\infty}^{\infty} U(t) dF(t) = \text{Expected Utility} \tag{2}$$

Bordley and LiCalzi (2000) build on this result and show that selecting an action that maximizes the probability of meeting an independent uncertain target (but whose value is revealed only after the manager’s performance is complete) is equivalent to maximizing expected utility.

Abbas and Matheson (2005) also discuss normative target-based formulations for single attributes and define the *aspiration equivalent* of a continuous and strictly monotonic probability distribution  $F(x)$  and utility function,  $U(x)$ , by the equation

$$\hat{x} \triangleq F^{-1} \left( \int_{-\infty}^{\infty} F(x) dU(x) \right) \tag{3}$$

that is analogous to the certain equivalent of a lottery,  $\tilde{x}$ ,

$$\tilde{x} \triangleq U^{-1} \left( \int_{-\infty}^{\infty} U(x) dF(x) \right) \tag{4}$$

From (3) and (4), and using the rule of integration by parts, we show that the certain equivalent and the aspiration equivalent are functionally related;

$$U(\tilde{x}) + F(\hat{x}) = 1 \tag{5}$$

Re-arranging (5) gives an interpretation for the expected utility of a lottery in terms of the aspiration equivalent as

$$U(\tilde{x}) = 1 - F(\hat{x}) \tag{6}$$

Equation (6) shows that the expected utility of a given lottery is equal to the probability that the outcome of the lottery exceeds the aspiration equivalent that is defined by (3). Thus, the aspiration equivalent provides a new interpretation for the expected utility of the lottery (Figure 3).

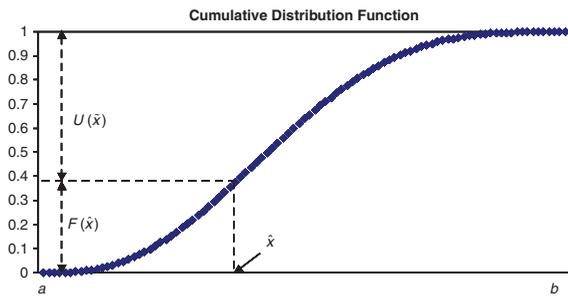


Figure 3. Expected utility is equal to the probability of exceeding the aspiration equivalent.

This result also provides us with a normative method for choosing between lotteries: *we choose the lottery that has the highest probability of meeting its aspiration equivalent*. The aspiration equivalent is thus a deterministic target that induces a manager who maximizes the probability of meeting (or exceeding) his target to choose the decision with the highest expected utility for the organization. Note that both the certain equivalent and the aspiration equivalent are functions of both the utility function and the probability distribution function under consideration. This means that the appropriate target must be set differently for each lottery under consideration. Simon speaks to this point when he discusses revising targets if they are too easy to achieve.

We have also found some background work on target-based formulations for multiple attributes. MBO is one such method that sets multiple independent targets for the managers but does not necessarily induce expected utility maximizing behaviour. To align this incentive structure with expected utility, Bordley and Kirkwood (2004) elegantly relate the expected utility to the probability of simultaneously meeting multiple independent targets over each attribute. Assuming probabilistic independence between the lotteries of the individual targets, they show that there always exists a multilinear utility function (Keeney and Raiffa, 1976) that is strategically equivalent to this target-based formulation. Their work provides a nice target-based interpretation for the expected utility of meeting multiple independent targets in terms of the well-known multilinear form. It is also a positive step in the direction of showing companies why arbitrary independent targets may not yield expected utility maximizing decisions. As we highlight in this paper, however, the target-based incentive

structure of meeting multiple independent targets that is widely used in organizational settings may still violate the value-based target requirement that we discussed above even if the probability of meeting the target is equal to the expected utility. Tsetlin and Winkler's (2006) work, which is also built on the premise of setting and simultaneously achieving multiple independent targets, derives impossibility conditions for multiattribute target-based formulations even for the simple case of continuous and bounded multiattribute utility functions that are strictly increasing with each of their arguments. These impossibility conditions arise because their formulation leads to multiattribute utility functions that violate reasonable value trade-off requirements. In this paper, we show how a decision maker can set a multiattribute target that matches both his expected utility and his value trade-offs. At the same time, we do not require probabilistic independence between the different target lotteries, and we do not require the utility function (or the value function) to be strictly monotonic with each of its arguments. We hope that this work will highlight the many drawbacks that are induced by the existing target-based incentive structures both in real organizational settings and in theoretical approaches.

### 3. ISSUES WITH SETTING INDEPENDENT PERFORMANCE TARGETS

We now consider multiattribute target-based settings that induce expected utility maximizing behaviour. While it might seem at first that the natural extension of the single-attribute aspiration equivalent (or target) formulation would be to set a fixed target for each attribute, thereby defining a rectangular region as the target area, we show that this target-setting approach may lead to decisions that are inconsistent with the decision maker's value trade-offs.

To illustrate, let us first refer back to deterministic multiattribute settings. In deterministic decision analysis, a decision maker first assigns a value function across the different attributes to choose between different alternatives. Usually the decision maker will have some tradeoffs (or marginal rates of substitution) from one attribute to another. For example, Howard (1984) uses a value function for a decision maker who is exposed to fates comparable to death (such as outcomes of medical surgery).

The decision maker provides a value function over two attributes: consumption,  $x$ , and health state,  $y$ . The value function over consumption and health states is given as

$$V(x, y) = xy^\eta \tag{7}$$

where  $x$  is expressed in dollars,  $y$  is the health state, and  $\eta$  determines the trade-off between  $x$  and  $y$ . This value function defines the isopreference curves illustrated in Figure 4. These contours show the decision makers the deterministic trade-offs.

The term  $\eta$  is a trade-off coefficient determined by calculating the fractional increase in attribute  $x$  compared with the fractional decrease in attribute  $y$  that makes the decision maker indifferent. That is on the same isopreference contour,  $C$ ,

$$\eta = -\left(\frac{dx}{x}\right) / \left(\frac{dy}{y}\right) \tag{8}$$

As the value of  $\eta$  approaches zero, it implies no substitution between the attributes (Figure 4(a)), and the isopreference contours define a rectangular region. As  $\eta$  increases, the trade-off contours allow for substitution among the attributes (Figure 4(b) shows the case where  $\eta = 0.7$ ). As  $\eta$  increases further, (Figure 4(c)) we have little substitution among the attributes. These trade-off contours provide a set of ordinal (deterministic) preferences as a first step toward building a cardinal value function.

When uncertainty is present, we assign a utility function over the value function to represent the decision maker's risk attitude

$$U(x_1, \dots, x_n) = U_V(V(x_1, \dots, x_n)) \tag{9}$$

where  $x_1, \dots, x_n$  are the attributes of interest,  $V(x_1, \dots, x_n)$  is the deterministic value function,  $U_V(V)$  is the utility on value, and the composite function,  $U(x_1, \dots, x_n)$ , is the multiattribute utility function (see Matheson and Abbas, 2005 for more details on constructing multiattribute utility functions with value functions). We refer to  $U_V(V)$  as the value-based utility. As the cardinal utility function is always a monotonic transformation of the (ordinal) value function, it has identical isopreference curves.

Now suppose an analyst asks the decision maker to set a target independently for each attribute by announcing,  $x_T$  and  $y_T$  as the target threshold values. As we discussed, these two values define the rectangular region in Figure 2. This formulation would be inconsistent with the decision maker's value trade-offs (Figure 5) unless he has no substitution among the attributes.

Moreover, by setting a target as the rectangular shaded region we find there are points outside this target area that are equally or more preferred to points in the target area. From a target-setting perspective, it would be counter-intuitive to define such a target as a goal if there are better outcomes outside the target region. We now illustrate how consistency can be achieved using our definition of a deterministic normative target that satisfies the value-based requirement.

#### 4. THE VALUE ASPIRATION EQUIVALENT

##### 4.1. Value-based definitions

We consider multiattribute utility functions with  $n$  attributes,  $X_1, X_2, \dots, X_n$ . We use the lower case,

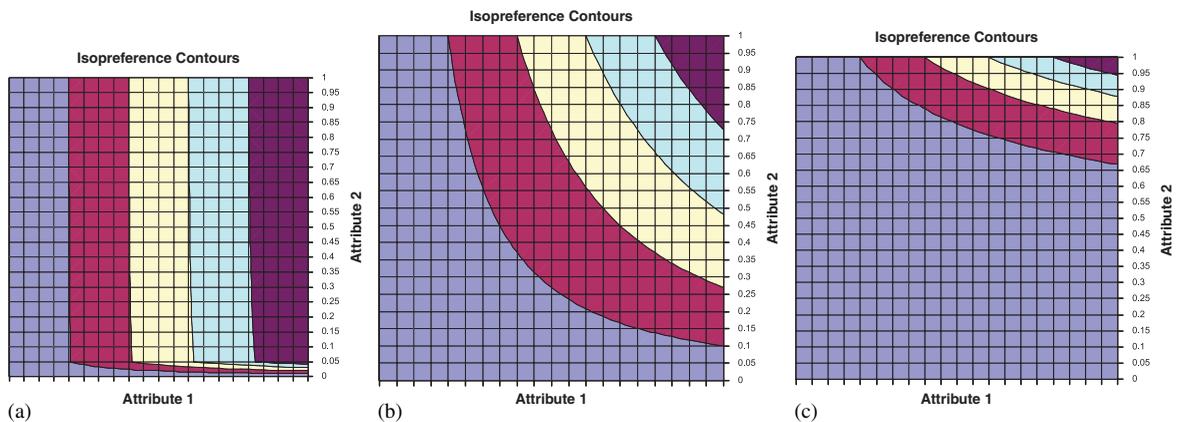


Figure 4. Isopreference contours: (a)  $\eta = 0.01$ , (b)  $\eta = 0.7$ , (c)  $\eta = 4$ .

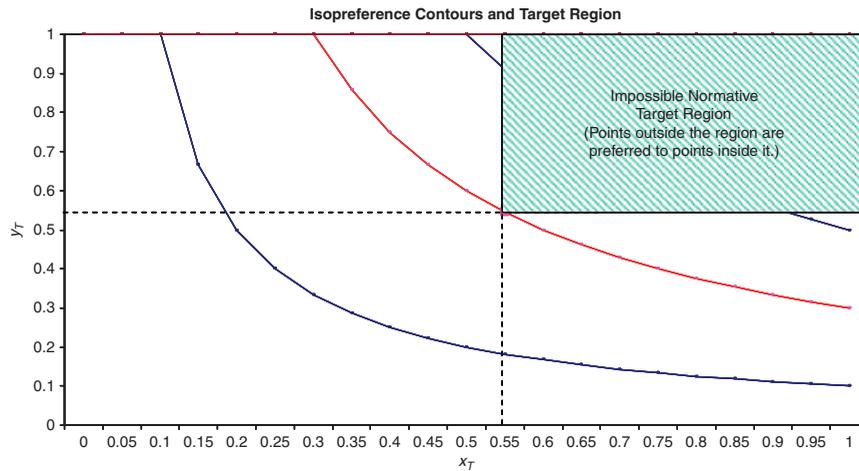


Figure 5. Shaded region shows a rectangular target region set by individual targets for each attribute when the actual isopreference contours allow for substitution. It would be quite unreasonable to set a rectangular target in this setting.

$x_i, i \in \{1, 2, \dots, n\}$  to denote a certain instantiation of attribute  $X_i$ , and use  $x_i^0$  and  $x_i^*$  to denote the minimum and maximum values of  $X_i$  (respectively). We use the vector of attributes,  $X = (x_1, \dots, x_n)$ , to represent a consequence of the decision situation defined on the domain  $[x_1^0, x_1^*] \times [x_2^0, x_2^*] \times \dots \times [x_n^0, x_n^*]$ . We assume that the multiattribute utility function,  $U(x_1, x_2, \dots, x_n)$ , is locally non-constant, and bounded. However, we do not assume that it is strictly increasing with each of its arguments. In fact it can even have multiple peaks. We use  $V(x_1, \dots, x_n)$  to represent the decision maker's deterministic value function over  $n$  attributes,  $x_1, \dots, x_n$ . We also define  $U_V(V)$  as the utility function over value (or the value-based utility). We use a normalized utility function over value such that

$$U_V(V_{\min}) = 0 \quad \text{and} \quad U_V(V_{\max}) = 1 \quad (10)$$

where  $V_{\min}$  and  $V_{\max}$  are the minimum and maximum values of the value function respectively. We assume also assume that  $U_V$  is continuous and strictly increasing with  $V$ .

We denote the joint cumulative probability distribution of a given alternative as  $F(x_1, x_2, \dots, x_n)$ , and assume that the joint probability density function  $f(x_1, x_2, \dots, x_n) = \frac{\partial^n}{\partial x_1 \dots \partial x_n} F(x_1, x_2, \dots, x_n)$  exists and is positive. We do not require probability independence among the variables of the joint distribution.

To define our multiattribute target, we now introduce the notion of a value-based probability, which reduces a joint probability distribution function into a 1-D probability function over the value function.

#### 4.2. Value-based cumulative probability function

We define the value-based cumulative probability distribution function,  $F(V_0)$ , as the probability that the outcome of a joint distribution (lottery) is less than or equal to  $V$ . For example, for any  $n$ -variable probability density function,  $f(x_1, x_2, \dots, x_n)$ , we define a value-based cumulative probability as

$$F(V_0) \triangleq \iiint_{V(x_1, x_2, \dots, x_n) \leq V_0} f(x_1, x_2, \dots, x_n) dx_1, dx_2, \dots, dx_n \quad (11)$$

#### 4.3. Value-certain equivalent

Having defined a value-based utility,  $U_V(V)$ , and a value-based probability,  $F(V)$ , the expected utility over value for a given alternative is given using these definitions by the integral

$$\text{Value Expected Utility} = \int_{F(V_{\min})}^{F(V_{\max})} U_V(V) dF(V) \quad (12)$$

The value certain equivalent is the value-based utility-inverse of the expected utility. That is

$$\begin{aligned} &\text{Value Certain Equivalent} \\ &= \tilde{V} = U_V^{-1}(\text{Value Expected Utility}) \quad (13) \end{aligned}$$

The value certain equivalent defines a whole isopreference surface: all points have the same value on that surface defined by

$$\{(x_1, \dots, x_n) : V(x_1, \dots, x_n) = \tilde{V}\} \quad (14)$$

The same certain equivalent is achieved by any point on the isopreference surface. Normative expected utility maximization is equivalent to choosing the alternative with the highest value certain equivalent. Note that the set of points defined by (14) can represent multiple contours if the value function has multiple peaks.

#### 4.4. Value-aspiration equivalent

We now define a new quantity that allows us to deal with normative multiattribute targets. We first define the value expected disutility for a given alternative as

$$\begin{aligned} &\text{Value Expected Disutility} \\ &= 1 - \text{Value Expected Utility} \end{aligned}$$

Using the rule of integration by parts, we get

$$\begin{aligned} &\text{Value Expected Disutility} \\ &= 1 - \int_{F(V_{\min})}^{F(V_{\max})} U_V(V) dF(V) \\ &= 1 - \left( U_V(V)F(V) \Big|_{V_{\min}}^{V_{\max}} - \int_{U_V(V_{\min})=0}^{U_V(V_{\max})=1} F(V) dU_V(V) \right) \\ &= 1 - \left( 1 - \int_{U_V(V_{\min})=0}^{U_V(V_{\max})=1} F(V) dU_V(V) \right) \\ &= \int_{U_V(V_{\min})=0}^{U_V(V_{\max})=1} F(V) dU_V(V) \quad (15) \end{aligned}$$

where  $U_V(V)$  is normalized to range from 0 to 1. Note that the positions of the terms  $U_V(V)$  and  $F(V)$  have been interchanged in the integrals (12) and the last line in (15).

Finally, we now define the magnitude of the value aspiration equivalent over the value

function as

$$\begin{aligned} &\text{Value Aspiration Equivalent} \\ &= \hat{V} \triangleq F^{-1}(\text{Value Expected Disutility}) \\ &= F^{-1} \left( \int_{U(V_{\min})=0}^{U(V_{\max})=1} F(V) dU_V(V) \right) \quad (16) \end{aligned}$$

If the value-based cumulative probability is increasing at the value expected disutility, then (16) uniquely defines the value aspiration equivalent. Like the value certain equivalent, the value aspiration equivalent defines a contour of points on the space of possible outcomes that have the same value as determined by the decision maker's value function. These points are defined by

$$\{(x_1, \dots, x_n) : V(x_1, \dots, x_n) = \hat{V}\} \quad (17)$$

Figure 6 shows an example of contours defining a value aspiration equivalent and value certain equivalent. The probability that the outcome of the multivariate lottery lies within the target region is equal to the expected utility of this multivariate lottery. We thereby interpret expected multiattribute utility as a probability of falling within this target region, and a probability of successfully achieving the target. The value aspiration equivalent also defines trade-offs among targets with multiple attributes. In this multiattribute setting, the value aspiration equivalent defines one (or more) isopreference surfaces that surround the target region(s). All points outside the target region are less preferred to all points inside it.

Using the value aspiration equivalent provides a deterministic multiattribute target that uniquely satisfies the normative and value-based properties and allows for explicit trade-offs between the individual attributes that are consistent with the decision maker's value function.

Note that the value certain equivalent and value aspiration equivalent are related by re-arranging (13) and (16) to get

$$U_V(\tilde{V}) + F(\hat{V}) = 1 \quad (18)$$

Therefore, the expected utility of the multiattribute lottery,  $U_V(\tilde{V})$ , is

$$U_V(\tilde{V}) = 1 - F(\hat{V}) \quad (19)$$

which is the probability that the outcome of the lottery lies within the region defined by the value aspiration equivalent isopreference contour. If the value aspiration equivalent is used to define

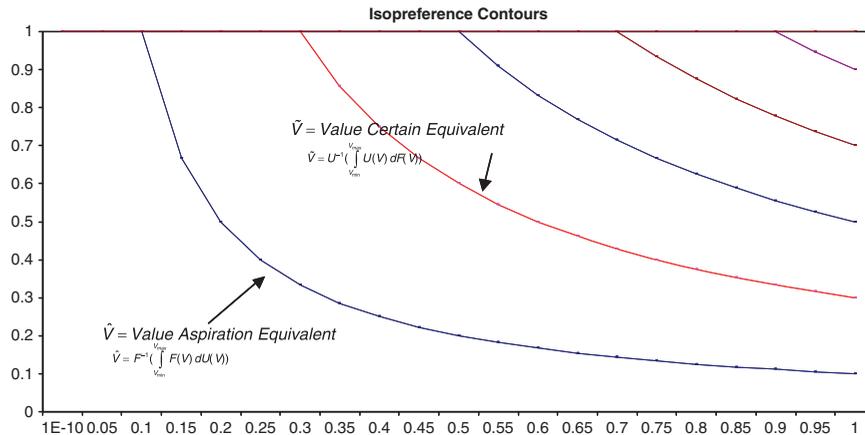


Figure 6. Value aspiration equivalents and value certain equivalents.

the multiattribute target region, then a decision maker who chooses the lottery that maximizes the probability of meeting his target also chooses the lottery that maximizes the expected utility.

**4.5. Example: setting normative multiattribute targets for price and profit margin**

In this example we demonstrate numerically how to set a normative and value-based multiattribute target. Let us assume that a company is interested in two attributes of a product: selling price of the product,  $p$ , and its profit margin,  $m$ . The first step is to determine the company's value function for the attributes. If the company is interested in profit, the value function is simply the product of the two attributes

$$V(p, m) = pm, \quad 0 \leq p \leq 1, \quad 0 \leq m \leq 1 \quad (20)$$

where the selling price,  $p$ , is normalized by dividing it by the maximum contemplated price,  $p_{max}$ , and  $m$  is the profit margin (fraction from 0 to 1).

Note that the value function has the following minimum and maximum values

$$V_{max} = 1, \quad V_{min} = 0 \quad (21)$$

If the company has an exponential utility function over value, then the normalized utility function is

$$U_V(V) = \frac{1 - e^{-\gamma V}}{1 - e^{-\gamma}} \quad (22)$$

where  $\gamma$  is the value risk aversion coefficient. For our example, let us assume the value of  $\gamma$  is 2.

For simplicity of the example, we will assume the joint probability density function of a

multiattribute lottery that the company is facing is uniformly distributed over the domain of the normalized attributes,  $0 \leq p \leq 1, 0 \leq m \leq 1$  and is given as

$$f(p, m) = 1, \quad 0 \leq p \leq 1, 0 \leq m \leq 1 \quad (23)$$

To facilitate the expressions, let  $p_1$  be the normalized price value where the indifference curves intersect the boundary  $m = 1$ . At this point  $V = p_1$ . Now we calculate the value-based cumulative probability for the isopreference curve which contains the point  $(p_1, m)$ .

$$\begin{aligned} F(V(p, m)) &= \int_{p_1}^1 \int_0^{\frac{V}{p}} 1 \, dm \, dp + p_1 \\ &= \int_{p_1}^1 \frac{V}{p} \, dp + p_1 \\ &= -V \ln(p_1) + p_1 \\ &= V(1 - \ln(V)) \\ &= pm(1 - \ln(pm)) \end{aligned} \quad (24)$$

Therefore, the value-based cumulative probability is

$$F(V) = V(1 - \ln(V)) \quad (25)$$

Figure 7 shows the value-based probability and utility functions for this example.

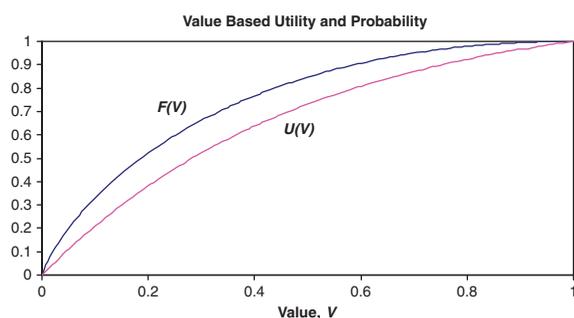


Figure 7. Value-based utility and probability.

By direct numerical calculation, we have

Value Expected Utility

$$= \int_{F(V_{\min})}^{F(V_{\max})} U_V(V) dF(V) = 0.38 \quad (26)$$

Value Expected Distility

$$= 1 - \text{Value Expected Utility} = 0.62 \quad (27)$$

Value Certain Equivalent =  $\tilde{V}$

$$= U_V^{-1}(0.38) = 0.198 \quad (28)$$

Value Aspiration Equivalent =  $\hat{V}$

$$= F^{-1}(0.62) = 0.27 \quad (29)$$

If the company uses the value aspiration equivalent as a normative multiattribute performance target, then the target should be specified as the isopreference curve

$$\{(p, m) : pm = 0.27\} \quad (30)$$

The target region itself is specified by the area:

$$\{(p, m) : pm \geq 0.27\} \quad (31)$$

Figure 8 shows the value aspiration equivalent and value certain equivalent for this example. The region above the value aspiration equivalent is the normative multiattribute target area for this lottery.

Substituting for the value of the aspiration equivalent into the value-based cumulative probability shows that the company has a probability of

$$F(0.27) = 0.27(1 - \ln(0.27)) = 0.62 \quad (32)$$

of not meeting its normative target with this uniform multiattribute lottery, and has a probability of 0.38 of exceeding its normative target with this lottery. Note further that the

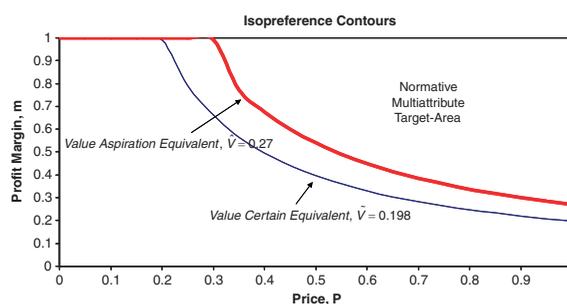


Figure 8. Normative multiattribute target.

probability of meeting the normative target is equal to the expected utility of the multiattribute lottery,  $U(\hat{x})$ , and the probability of not meeting it is equal to the expected disutility,  $F(\hat{x})$ . Thus if the company sets normative multiattribute targets for each alternative, then a manager who chooses his lotteries to maximize the probability of meeting its normative target (the value aspiration equivalent), will also choose the lottery with the highest expected utility for the organization. If the company specifies any other contour besides the value aspiration equivalent as a target, then a manager who is rewarded by meeting his targets may choose alternatives that do not maximize the corporation's expected utility.

As shown in Figure 8, there is no region outside the normative multiattribute target area that is more preferred than a region within it. Furthermore, Equation (30) determines the trade-offs between the attributes at the target values. For example, to meet a constant value the derivative of (30) is set equal to zero and we have

$$p dm + m dp = 0 \quad (33)$$

Re-arranging (33) gives the explicit trade-offs between the performance outcomes that are induced by the value aspiration equivalent target as

$$\frac{dp}{dm} = -\frac{p}{m} \quad (34)$$

From (34), a decision maker can make trade-offs between multiattribute outcomes that are consistent with the firm's value trade-offs.

#### 4.6. Example: utility functions that are not strictly increasing with each of their arguments

The use of the value aspiration equivalent is not limited to multiattribute utility functions that are strictly increasing with each of their arguments.

To illustrate consider the following example for constructing a multiattribute utility function of a peanut butter and jelly sandwich (Abbas and Howard, 2005). The attributes involved are thickness of both slices of bread, thickness of peanut butter, thickness of jelly, and the fraction of thickness of peanut butter to jelly.

Our preference for the sandwich is not a monotonically increasing function of the increase in thickness of peanut butter, jelly, or bread, and our preference for the thickness of jelly may change with the thickness of peanut butter. Nevertheless, we can construct a value function that determines our preference for the sandwich given any values of these four attributes. An example of a value function that returns a dollar amount for the peanut butter and jelly sandwich is shown below,

$$V(p, j, b, f) = V_{\max} \frac{pjbf}{(p^* j^* b^* f^*)^2} (2p^* - p)(2j^* - j) \times (2b^* - b)(2f^* - f) \quad (35)$$

where  $p, j,$  and  $b$  are the thicknesses of peanut butter, jelly, and bread respectively;  $p^*, j^*,$  and  $b^*$  are their optimal values,  $f$  is the fraction of thickness of peanut butter to jelly;  $f^*$  is the optimal fraction, and  $V_{\max}$  is the dollar amount that the decision maker is willing to pay for the optimal sandwich. For the purposes of a graphical representation, let  $V_{\max} = \$3, b^* = 1, j^* = .3, p^* = .3, f^* = 1.5$ .

To construct the multiattribute utility function, we now assign a 1-D utility function over value.

Figure 9 shows the multiattribute utility function constructed using a normalized exponential utility coefficient,  $\gamma = 0.01$ .

$$U_V(V) = \frac{e^{-\gamma V} - e^{-\gamma V_{\max}}}{e^{-\gamma V_{\min}} - e^{-\gamma V_{\max}}} \quad (36)$$

For the purposes of a 3-D representation of the utility surface and a 2-D contour representation, we fix  $j = .2$  and plot the multiattribute utility surface and isopreference contours as we vary  $b$  and  $p$ .

The contours of Figure 9 highlight the problem of setting independent targets, leading to a rectangular target region in the top right corner, when the peak of the value function occurs at the centre. This mismatch would be a major problem if set in practice. We can still use the value aspiration equivalent to set a normative target for this problem. By calculating the expected utility of a lottery, we identify the required target contour,  $C$ , using the value aspiration equivalent.

$$\begin{aligned} \text{Expected Utility} &= \int_{j_{\min}}^{j_{\max}} \int_{x_{\min}}^{x_{\max}} f(x, y) U(x, y) dx dy \\ &= \int_{V_{\min}}^{V_{\max}} U_V(V) dF(V) \end{aligned} \quad (37)$$

$$\text{Expected Distility} = 1 - \text{Expected Utility} \quad (38)$$

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$$= \hat{V} = F^{-1}(\text{Expected Distility}) \quad (39)$$

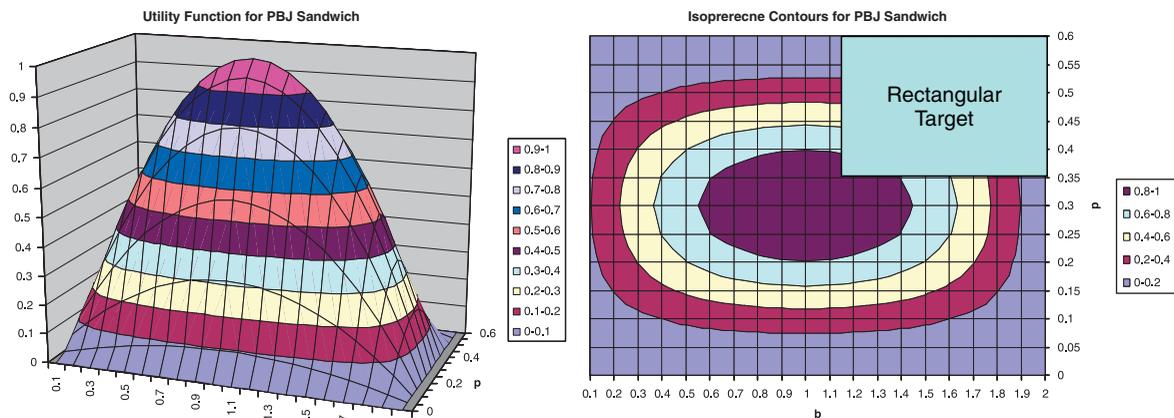


Figure 9. Multiattribute utility surface and isopreference contours for the peanut butter and jelly sandwich.

The target region is defined by the contour

$$(p, j, b, f) : V(p, j, b, f) = \hat{V}$$

from which trade-offs among the target objectives can be specified. The target itself satisfies

$$\{(p, j, b, f) : V(p, j, b, f) \geq \hat{V}\}$$

which defines parabolic contours in Figure 9.

## 5. CONCLUSIONS

The extension of Simon's notion of satisficing to decisions with multiple attributes is not to set multiple independent aspiration levels for each attribute or goal. Similarly, target-based formulations that extend to multiple objectives should not set multiple independent targets for each goal. We presented the concept of a deterministic multiattribute target region satisfying two fundamental properties (1) Value Consistency—there is no point outside the target region that is preferred to a point within the target and (2) Utility Consistency—the probability that the outcome of a multiattribute lottery lies within the target region is equal to the expected utility of the multiattribute lottery. In so doing, we provided an interpretation of the expected utility of a multiattribute lottery in terms of probability of success. We then defined the value aspiration equivalent that uniquely satisfies these two properties when the joint probability density is positive. We showed that the value aspiration equivalent provides a method for conceptualizing the expected utility of a multiattribute lottery in terms of probability of success and maintains trade-offs in correspondence with the decision maker's value function. We also showed that this formulation does not require monotonicity of the utility function or probabilistic independence of the multiattribute target lotteries.

On a more fundamental level, do we need to set targets to make better decisions? No. For a decision analyst, maximizing certain equivalents works fine. However, many others, dating back at least to Simon, are interested in setting targets (or goals). For them, we have shown a normative way to set fixed-threshold (deterministic) multiattribute targets. We have also shown that such deterministic normative targets must vary with the lottery under consideration for the same reasons that the value certain equivalent also

varies with the lottery under consideration. For example, the target set for an oil-well exploration project will be different than a target set for oil-well development, and a target set for a new exploratory drug will be different than a target set for an enhancement in an existing drug. In addition, these deterministic targets are bounded by the isopreference contours to introduce value consistency.

Arbitrarily choosing targets can result in decisions that are inconsistent with normative behaviour. The practice of setting individual targets for each attribute will usually be inconsistent with the value function trade-offs. The literature that advocates setting multiple independent targets, such as management by objectives, is advocating a non-normative method.

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